The Mathematics Teacher Preparation Competencies identify the essential knowledge and skills that must be developed and assessed in preparation programs that result in eligibility for the following Louisiana teaching certifications:

- Fourth – Eighth Grade Mathematics (4-8)
- Secondary Mathematics (Sixth – Twelfth Grade) (6-12)

Throughout this document, "student" is inclusive of and equally emphasizes students with exceptionalities, students from diverse language backgrounds, students designated as "high achieving," students at risk of academic failure, and students without exceptionalities.

Types of Competencies

These competencies include **content knowledge competencies** and **teaching competencies**. The content knowledge competencies identify foundational knowledge of mathematics. The teaching competencies identify teaching knowledge and skills that are specific to mathematics instruction.

Development of Competencies

These competencies were informed by Louisiana's academic standards and are aligned with national teacher preparation standards, including National Council for Teachers of Mathematics.

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MATHEMATICAL CONTENT KNOWLEDGE

I. Number and Quantity: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to numbers, number systems, and quantities incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

I.1: Candidates explain the place value system including:

I.1a: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and one-tenth of what it represents in the place to its left.

I.1b: Explain and apply patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain and apply patterns in the values of the digits in the product or the quotient, when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

I.1c: Read, write, and compare decimals to thousandths.

i. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).
ii. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

I.1d: Use place value understanding to round decimals to any place.

I.2: Candidates use place value understanding and properties of operations to perform multi-digit arithmetic.

I.3: Candidates fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

I.4: Candidates understand fractions as numbers including:

I.4a: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

I.4b: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

i. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
ii. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

I.4c: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line and explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two

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fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

I.4d: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

I.5: Candidates apply and extend previous understandings of operations with whole numbers to operations with fractions including:

I.5a: Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

i. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
ii. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

I.5b: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

I.5c: Understand a fraction a/b as a multiple of $1/b$ and understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.

I.5d: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

I.5e: Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

I.5f: Interpret multiplication as scaling (resizing), by:

i. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
ii. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (nxa)/(nxb)$ to the effect of multiplying a/b by 1.

I.5g: Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

I.5h: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

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I.6: Candidates apply and extend previous understandings of numbers to the system of rational numbers. including:

I.6a: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

I.6b: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

i. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
ii. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
iii. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

I.6c: Understand ordering and absolute value of rational numbers.

i. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
iii. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
iv. Distinguish comparisons of absolute value from statements about order.

I.7: Candidates apply and extend previous understandings of operations with fractions to operations with rational numbers including:

I.7a: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

i. Describe situations in which opposite quantities combine to make 0.
ii. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
iii. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

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iv. Apply properties of operations as strategies to add and subtract rational numbers.

I.7b: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

i. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
ii. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real- world contexts.
iii. Apply properties of operations as strategies to multiply and divide rational numbers.
iv. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

I.7c: Solve real-world and mathematical problems involving the four operations with rational numbers

I.8: Candidates use properties of rational and irrational numbers including:

I.8a: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

I.8b: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

I.8c: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

I.9: Candidates work with radicals and rational exponents including:

I.9a: Know and apply the properties of integer exponents to generate equivalent numerical expressions.

I.9b: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

I.9c: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

I.10: Candidates reason quantitatively and use units to solve problems including:

I.10a: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

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I.10b: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

I.10c: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

I.10d: Define appropriate quantities for the purpose of descriptive modeling.

I.10e: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

I.11: Candidates perform arithmetic operations with complex numbers including:

I.11a: Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

I.11b: Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

I.11c: Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

I.12 for 6-12: Candidates represent complex numbers and their operations on the complex plane including:

I.11a: Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

I.11b: Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

I.11c: Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

I.13 for 6-12: Candidates use complex numbers in polynomial identities and equations including:

I.13a: Solve quadratic equations with real coefficients that have complex solutions.

I.13b: Extend polynomial identities to the complex numbers.

I.13c: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

I.14 for 6-12: Candidates represent and model with vector quantities including:

I.14a: Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, | ***v*** |, || ***v*** ||, ***v***).

I.14b: Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

I.14c: Solve problems involving velocity and other quantities that can be represented by vectors.

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I.15 for 6-12: Candidates perform operations on vectors including:

I.15a: Add and subtract vectors.

i. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

ii. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

iii. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

I.15b: Multiply a vector by a scalar.

i. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

ii. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

I.16 for 6-12: Candidates perform operations on matrices and use matrices in applications including:

I.16a: Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

I.16b: Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

I.16c: Add, subtract, and multiply matrices of appropriate dimensions.

I.16d: Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

I.16e: Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

I.16f: Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

I.16g: Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

II. Algebra: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to expression, equations and inequalities, and the connections to functions and modeling incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

II.1: Candidates understand and apply properties of operations and the relationship between addition and subtraction and between multiplication and division including:

II.1a: Apply properties of operations as strategies to add and subtract.

II.1b: Understand subtraction as an unknown-addend problem.

II.1c: Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

II.1d: Apply properties of operations as strategies to multiply and divide.

II.1e: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

II.1f: Understand division as an unknown-factor problem.

II.2: Candidates generate and analyze patterns including:

II.2a: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

II.2b: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

II.2c: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

II.3: Candidates interpret the structure of expressions including:

II.3a: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

II.3b: Interpret expressions that represent a quantity in terms of its context.

i. Interpret parts of an expression, such as terms, factors, and coefficients.
ii. Interpret complicated expressions by viewing one or more of their parts as a single entity.

II.3c: Use the structure of an expression to identify ways to rewrite it and understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

II.4: Candidates write expressions in equivalent forms to solve problems including:

II.4a: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

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II.4b: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

i. Factor a quadratic expression to reveal the zeros of the function it defines.
ii. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
iii. Use the properties of exponents to transform expressions for exponential functions.

II.4c (6-12 only): Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

II.5 for 6-12: Candidates understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

II.6 for 6-12: Candidates understand the relationship between zeros and factors of polynomials including:

II.6a: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

II.6b: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

II.7 for 6-12: Candidates use polynomial identities to solve problems including:

II.7a: Prove polynomial identities and use them to describe numerical relationships.

II.7b: Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

II.8 for 6-12: Candidates rewrite rational expressions including:

II.8a: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

II.8b: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

II.9: Candidates understand solving equations as a process of reasoning and explain the reasoning including:

II.9a: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

II.9b: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

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II.9c: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

II.10: Candidates solve equations and inequalities in one variable including:

II.10a: Solve linear equations in one variable.

i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

II.10b: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

II.10c: Solve quadratic equations in one variable.

i. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2$ = q that has the same solutions. Derive the quadratic formula from this form.
ii. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

II.11: Candidates solve systems of equations including:

II.11a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

II.11b: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

II.11c: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

II.11d (6-12 only): Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

II.11e (6-12 only): Represent a system of linear equations as a single matrix equation in a vector variable.

II.11f (6-12 only): Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

II.12: Candidates represent and solve equations and inequalities graphically including:

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II.12a: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

II.12b: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

II.12c: Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

II.13: Candidates create equations that describe numbers or relationships including:

II.13a: Create equations and inequalities in one variable and use them to solve problems.

II.13b: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

II.13c: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context.

II.13d: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

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III. Functions: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to functions and the connections to expressions, equations, modeling, and coordinates incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

III.1: Candidates understand ratio concepts and use ratio reasoning to solve problems including:

III.1a: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

III.1b: Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship.

III.1c: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

i. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
ii. Solve unit rate problems including those involving unit pricing and constant speed.
iii. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
iv. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

III.2: Candidates analyze proportional relationships and use them to solve real-world and mathematical problems including:

III.2a: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

III.2b: Recognize and represent proportional relationships between quantities.

i. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
ii. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
iii. Represent proportional relationships by equations.
iv. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

III.2c: Use proportional relationships to solve multistep ratio and percent problems.

III.2d: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

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III.3: Candidates understand the concept of a function and use function notation including:

III.3a: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

III.3b: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

III.3c: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

III.4: Candidates use functions to model relationships between quantities and interpret functions that arise in applications in terms of the context including:

III.4a: Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

III.4b: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

III.4c: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

III.4d: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

III.4e: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

III.4f: Interpret the parameters in a linear or exponential function in terms of a context.

III.5 for 6-12: Candidates analyze functions using different representations including:

III.5a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

i. Graph linear and quadratic functions and show intercepts, maxima, and minima.
ii. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
iii. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
iv. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
v. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

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III.5b: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

i. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
ii. Use the properties of exponents to interpret expressions for exponential functions.

III.5c: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

III.6 for 6-12: Candidates build a function that models a relationship between two quantities including:

III.6a: Write a function that describes a relationship between two quantities.

i. Determine an explicit expression, a recursive process, or steps for calculation from a context.
ii. Combine standard function types using arithmetic operations.
iii. Compose functions.

III.6b: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

III.7 for 6-12: Candidates build new functions from existing functions including:

III.7a: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

III.7b: Find inverse functions.

i. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
ii. Verify by composition that one function is the inverse of another.
iii. Read values of an inverse function from a graph or a table, given that the function has an inverse.
iv. Produce an invertible function from a non-invertible function by restricting the domain.

III.7c: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

III.8 for 6-12: Candidates construct and compare linear, quadratic, and exponential models and solve problems including:

III.8a: Distinguish between situations that can be modeled with linear functions and with exponential functions.

i. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
ii. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

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iii. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

III.8b: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

III.8c: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

III.8d: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

III.9 for 6-12: Candidates extend the domain of trigonometric functions using the unit circle including:

III.9a: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

III.9b: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

III.9c: Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

III.9d: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

III.10 for 6-12: Candidates model periodic phenomena with trigonometric functions including:

III.10a: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

III.10b: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

III.10c: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

III.11 for 6-12: Candidates model periodic phenomena with trigonometric functions including:

III.11a: Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.

III.11b: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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IV. Geometry - Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to geometry, measurement, and trigonometry incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

IV.1: Candidates reason with shapes and their attributes including:

IV.1a: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

IV.1b: Understand that attributes belonging to a category of two- dimensional figures also belong to all subcategories of that category.

IV.1c: Classify two-dimensional figures in a hierarchy based on properties.

IV.1d: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

IV.1e: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

IV.1f: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

IV.1g: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

IV.2: Candidates solve problems involving measurement and conversion of measurements including:

IV.2a: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

IV.2b: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

IV.3: Candidates solve real-life and mathematical problems involving perimeter, area, and surface area including:

IV.3a: Recognize area as an attribute of plane figures and understand concepts of area measurement.

i. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
ii. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

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IV.3b: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

IV.3c: Relate area to the operations of multiplication and addition.

i. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
ii. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
iii. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
iv. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

IV.3d: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

IV.3e: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

IV.3f: Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

IV.3g: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

IV.4: Candidates solve real-life and mathematical problems involving volume including:

IV.4a: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

i. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
ii. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

IV.4b: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

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IV.4c: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

i. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
ii. Apply the formulas *V = l x w x h* and *V = b x h* for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real world and mathematical problems.
iii. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

IV.4d: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas *V = l w h* and *V = b h* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

IV.4e: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

IV.4f: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

IV.4g (6-12 only): Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

IV.4h (6-12 only): Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

IV.5: Candidates solve real-life and mathematical problems involving angles and angle measure including:

IV.5a: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

i. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
ii. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

IV.5b: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

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IV.5c: Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

IV.5d: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

IV.5e: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

IV.6: Candidates experiment with transformations in the plane including:

IV.6a: Verify experimentally the properties of rotations, reflections, and translations:

i. Lines are taken to lines, and line segments to line segments of the same length.
ii. Angles are taken to angles of the same measure.
iii. Parallel lines are taken to parallel lines.

IV.6b: Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

IV.6c: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

IV.6d: Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

IV.6e: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

IV.6f: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

IV.7: Candidates understand congruence in terms of rigid motions including:

IV.7a: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

IV.7b: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

IV.7c: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

IV.7d: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

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IV.7e: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

IV.8: Candidates prove geometric theorems involving congruence and similarity about lines and angles, triangles, and parallelograms.

IV.9: Candidates draw, construct, and describe geometrical figures and describe the relationships between them including:

IV.9a: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

IV.9b: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

IV.9c: Identify the shapes of two-dimensional cross-sections of three- dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

IV.9d: Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

IV.9e: Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

IV.10: Candidates understand similarity in terms of similarity transformations including:

IV.10a: Verify experimentally the properties of dilations given by a center and a scale factor:

i. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
ii. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

IV.10b: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them.

IV.10c: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

IV.10d: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

IV.10e: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

IV.11: Candidates use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

IV.12: Candidates define trigonometric ratios and solve problems involving right triangles including:

IV.12a: Explain a proof of the Pythagorean Theorem and its converse.

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IV.12b: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

IV.12c: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

IV.12d: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

IV.12e: Explain and use the relationship between the sine and cosine of complementary angles.

IV.12f: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

IV.13 for 6-12: Candidates apply trigonometry to general triangles including:

IV.13a: Derive the formula A = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

IV.13b: Prove the Laws of Sines and Cosines and use them to solve problems.

IV.13c: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

IV.14 for 6-12: Candidates understand and apply theorems about circles including:

IV.14a: Prove that all circles are similar.

IV.14b: Identify and describe relationships among inscribed angles, radii, and chords.

IV.14c: Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

IV.14d: Construct a tangent line from a point outside a given circle to the circle.

IV.15 for 6-12: Candidates translate between the geometric description and the equation for a conic section including:

IV.15a: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

IV.15b: Derive the equation of a parabola given a focus and directrix.

IV.15c: Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

IV.16: Candidates use coordinates to prove simple geometric theorems algebraically including:

IV.16a: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

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IV.16b: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

IV.16c: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

IV.16d (6-12 only): Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

IV.16e (6-12 only): Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

IV.16f (6-12 only): Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

IV.17 for 6-12: Candidates apply geometric concepts in modeling situations including:

IV.17a: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

IV.17b: Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

IV.17c: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

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V. Statistics and Probability: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to statistics and probability and the connections to functions and modeling incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

V.2: Candidates summarize, represent, and interpret data on a single count or measurement variable including:

V.2a: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

V.2b: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

V.2c: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

V.2d: Represent data with plots on the real number line (dot plots, histograms, and box plots).

V.2e: Summarize numerical data sets in relation to their context, such as by:

i. reporting the number of observations
ii. describing the nature of the attribute under investigation, including how it was measured and its units of measurement
iii. giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered
iv. relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

V.2f: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

V.2g: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

V.2h: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

V.2i: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

V.3: Candidates summarize, represent, and interpret data on two categorical and quantitative variables including:

V.3a: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct a two-way table on two categorical variables. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

V.3b: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

V.3c: Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

V.3d: Informally assess the fit of a function by plotting and analyzing residuals.

V.4: Candidates interpret linear models including:

V.4a: Know that straight lines are widely used to model relationships between two quantitative variables.

V.4b: Fit a linear function for a scatter plot that suggests a linear association.

V.4c: Use the equation of a linear model to solve problems in the context of bivariate measurement data.

V.4d: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

V.4e: Compute (using technology) and interpret the correlation coefficient of a linear fit.

V.4f: Distinguish between correlation and causation.

V.5: Candidates use random sampling to draw inferences about a population including:

V.5a: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

V.5b: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

V.6 for 6-12: Candidates Understand and evaluate random processes underlying statistical experiments including:

V.6a: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

V.6b: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

V.7 for 6-12: Candidates make inferences and justify conclusions from sample surveys, experiments, and observational studies:

V.7a: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

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V.7b: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

V.7c: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

V.7d: Evaluate reports based on data.

V.8 Candidates investigate chance processes and develop, use, and evaluate probability models including:

V.8a: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

V.8b: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

V.8c: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

i. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
ii. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

V.8d: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

i. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
ii. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
iii. Design and use a simulation to generate frequencies for compound events.

V.9 for 6-12: Candidates understand independence and conditional probability and use them to interpret data including:

V.9a: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

V.9b: Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

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V.9c: Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

V.9d: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

V.9e: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

V.10 for 6-12: Candidates use the rules of probability to compute probabilities of compound events in a uniform probability model including:

V.10a: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

V.10b: Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
V.10c: Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

V.10d: Use permutations and combinations to compute probabilities of compound events and solve problems.

V.11 for 6-12: Candidates use the rules of probability to compute probabilities of compound events in a uniform probability model including:

V.11a: Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

V.11b: Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

V.11c: Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

V.11d: Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

V.12 for 6-12: Candidates use probability to evaluate outcomes of decisions including:
V.12a: Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

i. Find the expected payoff for a game of chance.
ii. Evaluate and compare strategies on the basis of expected values.

V.12b: Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

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V.12c: Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

VI. Calculus: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to calculus incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

VI.1: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to limits, continuity, rates of change, the Fundamental Theorem of Calculus, and the meanings and techniques of differentiation and integration.

VI.2 for 6-12: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to parametric, polar, and vector functions.

VI.3 for 6-12: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to sequences and series.

VI.4 for 6-12: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to multivariate functions.

VI.5: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to applications of function, geometry, and trigonometry concepts to solve problems involving calculus.

VI.6: Candidates explain the historical development and perspectives of calculus including contributions of significant figures and diverse cultures.

VII. Discrete Mathematics for 6-12: To be prepared to develop learner mathematical proficiency, all candidates should know the following topics related to discrete mathematics with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

VII.1: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to discrete structures including sets, relations, functions, graphs, trees, and networks.

VII.2: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to enumeration including permutations, combinations, iteration, recursion, and finite differences.

VII.3: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to propositional and predicate logic.

VII.4: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to applications of discrete structures such as modeling and designing data structures.

VII.5: Candidates explain the historical development and perspectives of discrete mathematics including contributions of significant figures and diverse cultures.

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MATHEMATICS TEACHING COMPETENCIES

VII. Process of Instruction: Candidates plan and create appropriate, sequential, and challenging learning opportunities, grounded in mathematics education research and based on grade-level academic standards, in which students are actively engaged in building new knowledge from prior knowledge and experiences.

VII.1: Candidates develop and implement grade level instructional activities, routines, and experiences that develop learners' conceptual understanding while also teaching procedural skills.

VII. 2: Candidates use various learner groupings to foster deeper acquisition of conceptual understanding, skill, and fluency, leading to greater learner independence with mathematical thinking.

VII.3: Candidates break down elements of mathematics to make its structures apparent to learners and effectively explain the meaning of procedures orally, in writing, and using real life experiences, manipulatives, models, and pictures/diagrams.

VII.4: Candidates appropriately sequence content for instruction and use applicable scaffolding and remediation exercises so that learners are able to meet on-level standards.

VII.5: Candidates design and select standards-based tasks using varied strategies (i.e. real life applications, manipulatives, models, diagrams/pictures) that present opportunities for instruction and assessment.

VII.6: Candidates apply mathematical content and pedagogical knowledge to select and use instructional tools such as manipulatives and physical models, drawings, virtual environments, spreadsheets, presentation tools, mathematics-specific technologies (e.g., graphing tools, interactive geometry software and, in grades 6-12, computer algebra systems, and statistical packages), and representations such as tape diagrams, arrays, area models, number bonds, visual fraction models, number lines, and real life scenarios.

VII.7: Candidates support students' mathematical language development and require learners to explain their mathematical understanding both in writing and orally through classroom conversations, using appropriate prompting and questioning to allow learners to refine their mathematical thinking and build upon one another's understandings.

VII.8 Candidates plan, design, and implement activities and tasks for all learners to demonstrate proficiency with the math practices. This instruction intersects with mathematical content and requires students to demonstrate these practices within and among mathematical domains.

VIII. Process of Assessment: Candidates plan, design, and implement appropriate assessments that are grounded in mathematics education research, are based on Louisiana Academic Standards, and require students to demonstrate competency in fluency, procedural skills, and conceptual understanding.

VIII.1: Candidates are able to select or design a range of ongoing classroom assessments (e.g., diagnostic, formal and informal, formative and summative, oral and written) which determine how well learners are able to demonstrate understanding of math concepts and procedures.

VIII.2: Candidates are able to determine trends in assessment results. They use these results to adjust instructional strategies, provide differentiated instructional supports, and determine and appropriately communicate the strengths and weaknesses in instructional strategies and student performance with colleagues, learners, and families.

VIII.3: Candidates are able to interpret learner errors, gaps, and inconsistencies in knowledge, skills, and mathematical reasoning; they use that information to plan future instruction, activities, and experiences for learners.

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