The Mathematics Teacher Preparation Competencies identify the essential knowledge and skills that must be developed and assessed in preparation programs that result in eligibility for the following Louisiana early childhood and elementary teaching certifications:

- Birth – Kindergarten (B-K)
- Pre-Kindergarten – Third Grade (PK-3)
- First – Fifth Grade (1-5)

Throughout this document, "student" is inclusive of and equally emphasizes students with exceptionalities, students from diverse language backgrounds, students designated as "high achieving," students at risk of academic failure, and students without exceptionalities.

Types of Competencies

These competencies include **content knowledge competencies** and **teaching competencies**. The content knowledge competencies identify foundational knowledge of mathematics. The teaching competencies identify teaching knowledge and skills that are specific to mathematics instruction.

Development of Competencies

These competencies were informed by Louisiana's academic standards and are aligned with national teacher preparation standards, including National Association for the Education of Young Children standards.

MATHEMATICS CONTENT KNOWLEDGE

I. Number and Quantity: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to numbers, number systems, and quantities incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

I.1: Candidates understand the place value system including:

I.1a: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

I.1b: Explain and apply patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain and apply patterns in the values of the digits in the product or the quotient, when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

I.1c: Read, write, and compare decimals to thousandths.

i. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).
ii. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

I.1d: Use place value understanding to round decimals to any place.

I.2: Candidates use place value understanding and properties of operations to perform multi-digit arithmetic.

I.3: Candidates fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

I.4: Candidates understand fractions as numbers including:

I.4a: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

I.4b: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

i. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
ii. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

I.4c: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line and explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two

fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

I.4d: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

I.5: Candidates apply and extend previous understandings of operations with whole numbers to operations with fractions including:

I.5a: Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

i. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
ii. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

I.5b: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

I.5c: Understand a fraction a/b as a multiple of $1/b$ and understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.

I.5d: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

I.5e: Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

I.5f: Interpret multiplication as scaling (resizing), by:

i. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
ii. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (nxa)/(nxb)$ to the effect of multiplying a/b by 1.

I.5g: Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

I.5h: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

I.6: Candidates apply and extend previous understandings of numbers to the system of rational numbers. including:

I.6a: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

I.6b: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

i. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
ii. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
iii. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

I.6c: Understand ordering and absolute value of rational numbers.

i. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
iii. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
iv. Distinguish comparisons of absolute value from statements about order.

I.7: Candidates apply and extend previous understandings of operations with fractions to operations with rational numbers including:

I.7a: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

i. Describe situations in which opposite quantities combine to make 0.
ii. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
iii. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

iv. Apply properties of operations as strategies to add and subtract rational numbers.

I.7b: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

i. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
ii. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real- world contexts.
iii. Apply properties of operations as strategies to multiply and divide rational numbers.
iv. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

I.7c: Solve real-world and mathematical problems involving the four operations with rational numbers

II. Algebra: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to expression, equations and inequalities, and the connections to functions and modeling incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

II.1: Candidates understand and apply properties of operations and the relationship between addition and subtraction and between multiplication and division including:

II.1a: Apply properties of operations as strategies to add and subtract.

II.1b: Understand subtraction as an unknown-addend problem.

II.1c: Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

II.1d: Apply properties of operations as strategies to multiply and divide.

II.1e: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

II.1f: Understand division as an unknown-factor problem.

II.2: Candidates generate and analyze patterns including:

II.2a: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

II.2b: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

II.2c: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

II.9: Candidates understand solving equations as a process of reasoning and explain the reasoning including:

II.9a: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

II.9b: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

III. Functions: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to functions and the connections to expressions, equations, modeling, and coordinates incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

III.1: Candidates understand ratio concepts and use ratio reasoning to solve problems including:

III.1a: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

III.1b: Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

III.1c: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

i. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
ii. Solve unit rate problems including those involving unit pricing and constant speed.
iii. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
iv. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

IV. Geometry: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to geometry, measurement, and trigonometry incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

IV.1: Candidates reason with shapes and their attributes including:

IV.1a: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

IV.1b: Understand that attributes belonging to a category of two- dimensional figures also belong to all subcategories of that category.

IV.1c: Classify two-dimensional figures in a hierarchy based on properties.

IV.1d: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

IV.1e: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

IV.1f: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

IV.1g: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

IV.2: Candidates solve problems involving measurement and conversion of measurements including:

IV.2a: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

IV.2b: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

IV.3: Candidates solve real-life and mathematical problems involving perimeter, area, and surface area including:

IV.3a: Recognize area as an attribute of plane figures and understand concepts of area measurement.

i. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
ii. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

IV.3b: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

IV.3c: Relate area to the operations of multiplication and addition.

i. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
ii. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

iii. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

iv. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

IV.3d: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

IV.3e: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

IV.3f: Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

IV.3g: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

IV.4: Candidates solve real-life and mathematical problems involving volume including:

IV.4a: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

i. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

ii. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

IV.4b: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

IV.4c: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

i. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

ii. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real world and mathematical problems.

iii. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

IV.4d: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

IV.4e: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

IV.4f: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

IV.5: Candidates solve real-life and mathematical problems involving angles and angle measure including:

IV.5a: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

i. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
ii. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

IV.5b: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

IV.5c: Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

IV.5d: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

IV.5e: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

IV.16: Candidates use coordinates to prove simple geometric theorems algebraically including:

IV.16a: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

IV.16b: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

IV.16c: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

V. Statistics and Probability: Candidates demonstrate conceptual understanding, procedural skill and fluency, and ability to solve real-world problems related to statistics and probability and the connections to functions and modeling incorporating, when appropriate, mathematical practices, appropriate technology, and varied representational tools, including concrete models.

V.1: Candidates represent and interpret data including:

V.1a: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

V.1b: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve simple put- together, take-apart, and compare problems, as well as, one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.

V.1c: Generate measurement data by measuring lengths of several objects, or by making repeated measurements of the same object, using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

V.1d: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions to solve problems involving information presented in line plots.

MATHEMATICS TEACHING COMPETENCIES

VII. Process of Instruction: Candidates plan and create appropriate, sequential, and challenging learning opportunities, grounded in mathematics education research and based on Louisiana Academic Standards, in which students are actively engaged in building new knowledge from prior knowledge and experiences.

VII.1: Candidates develop and implement age- and grade-level instructional activities, routines, and experiences that develop students' conceptual understanding while also teaching procedural skills.

VII. 2: Candidates use various groupings (whole group when appropriate, small group and individualized) and free choice activities to foster deeper acquisition of conceptual understanding, skill and fluency, leading to greater independence with mathematical thinking.

VII.3: Candidates break down elements of mathematics to make its structures apparent to students and effectively explain the meaning of procedures orally, in writing, and using real life experiences, manipulatives, models, and pictures/diagrams.

VII.4: Candidates appropriately sequence content for instruction and use applicable scaffolding and remediation exercises so that students are able to meet on-level standards.

VII.5: Candidates design and select standards-based tasks using varied strategies (i.e. real life applications, manipulatives, models, diagrams/pictures) that present opportunities for instruction and assessment.

VII.6: Candidates apply mathematical content and pedagogical knowledge in the selection, use, and promotion of instructional tools such as manipulatives and physical models, drawings, virtual environments, presentation tools, mathematics-specific technologies (e.g., graphing tools and interactive geometry software), and representations such as tape diagrams, arrays, area models, number bonds, visual fraction models, number lines, and real life scenarios.

VII.7: Candidates support students' mathematical language development and require students to explain their mathematical understanding both in writing and orally through classroom conversations, using appropriate prompting and questioning to allow students to refine their mathematical thinking and build upon one another's understandings.

VII.8 Candidates plan, design, and implement activities and tasks for all students to demonstrate proficiency with the math practices. This instruction intersects with mathematical content and requires students to demonstrate these practices within and among mathematical domains.

VIII. Process of Assessment: Candidates plan, design, and implement appropriate assessments that are grounded in mathematics education research, are based on Louisiana Academic Standards, and require students to demonstrate competency in fluency, procedural skills, and conceptual understanding.

VIII.1: Candidates are able to select or design a range of ongoing classroom assessments (e.g., diagnostic, formal and informal, formative and summative, oral and written) which determine how well students are able to demonstrate understanding of math concepts and procedures.

VIII.2: Candidates are able to determine trends in assessment results. They use these results to adjust instructional strategies, provide differentiated instructional supports, and determine and appropriately communicate the strengths and weaknesses in instructional strategies and student performance with colleagues, students, and families.

VIII.3: Candidates are able to interpret student errors, gaps, and inconsistencies in knowledge, skills, and mathematical reasoning; they use that information to plan future instruction, activities, and experiences for students.